FRANCIS CORBLIN

EXISTENCE, MAXIMALITY, AND THE SEMANTICS OF NUMERAL MODIFIERS

Abstract. This paper provides a new proposal for the semantics of *at least* and *at most*. This proposal takes these expressions as comparing two sets, the maximal set of individuals satisfying the NP & VP conditions, and a set of individuals satisfying the NP constraints. This approach assumes that these expressions introduce both sets in the semantic representation. Data involving anaphora (some of them initially introduced in Kadmon 1987) and new data involving apposition, are presented as arguments supporting this claim. The paper gives the representation of the meaning of *at least* and *at most* induced by this approach in a DRT framework.

1. INTRODUCTION*

This paper is about the semantics of complex expressions like *at least two books, at most two books, exactly two books*, and its relation to the semantics of noun phrases involving a bare numerals (*one book, two books*). The starting point is N. Kadmon's (1987) observations on discourse anaphora to these two kinds of antecedents, and especially what will be called in this paper the "maximalization effect" of expressions like*at least.*

Kadmon observes that in (1) the discourse pronoun *they* must refer to a set of (exactly) ten kids, although in (2) in the preferred reading of *they*, it denotes the maximal sets of kids walking in the room:

(1) Ten kids walked into the room.
They were making an awful lot of noise.

(2) At least ten kids walked into the room.
They were making an awful lot of noise.

I will also consider in this paper some new data involving apposition, i.e. sentences like (3) and show that they confirm Kadmon's observations.

*I am very grateful to Bart Geurts, Ora Matushansky, and Gennaro Chierchia for their comments on previous presentations. I have also greatly benefited from the comments of three anonymous reviewers of the paper. The first presentation of this material took place in 2002, in the Nancy workshop *Existence: Semantics and Syntax*. My main objective in this contribution is to present and motivate the "two-set" theory of *at least/at most* introduced in this workshop. A full discussion of all alternative proposals, including later works, would have exceed the imparted limits.

I. Comorovski and K. von Heusinger (eds.), Existence: Semantics and Syntax, 223–252.

(3) I quoted at least two papers, Lewis (1975) and Cooper (1979).

The claim will be that in order to accommodate the relevant data it is necessary to assume that complex expressions like *at least two books* introduce two sets into the discourse:

- a set having the cardinality expressed by the number;
- the maximal set of individuals satisfying the conditions expressed by the sentence.

Pursuing a line of investigation initiated in previous works (Corblin 1997, 2002), I will sketch a proposal which takes the "maximalization effect" as a direct product of the common semantics of the terms *at least, at most, exactly.* This proposal takes expressions like *at least n Ns* as expressing a relation between two sets: a set of cardinality *n* (exactly) and the maximal set of individuals satisfying the descriptive content of the modified noun phrase and the properties expressed by the sentence.

This view is in line with Krifka's (1999) analysis, which argues that *at least* is neither a determiner nor an expression building a complex determiner in composition with a numeral, but an expression taking scope over the whole modified noun phrase, which may lack a determiner, as in (4), or a numeral, as in (5):

(4) She invited at least John and Mary.

(5) At least some determiners are not determiners. (Krifka 1999)

In contrast to Krifka (1999), the proposal does not rely on the sensitivity to focus of these expressions and consequently, alternatives do not play any role in the analysis.

Although the solution introduced in this paper tries to accommodate Kadmon's (1987) insights, it differs from Kadmon's proposal in two important ways: it does not correlate a syntactic ambiguity of *at least* to the introduction of two sets, and it does not defer a role to the interpretation of the anaphoric pronoun. In my view, it is the semantics of expressions like *at least* themselves that is responsible for introducing the two sets, and a plural pronoun will simply have a choice, in principle, between a reference to the maximal set and a reference to a set of exactly *n* elements.

The proposal thus diverges also from Landman (2000) which takes "numeral modifiers" as forming complex determiners with numerals and "introducing cardinality relations (relations between numbers)" (Landman 2000: 239).

Moreover, while most approaches tend to bring expressions like *at least* closer to true comparatives (*at least three = more than two*), the present work shows that too many empirical properties oppose the two constructions for them to be considered mere variants of a single category.

This makes, consequently, the usual terminology "numeral modifier", or "complex determiner" impractical and rather misleading for the present analysis of the expressions *at least*, *at most*, and *exactly.*

As a working terminology for the sake of this paper, I will use the following convention, which preserves as far as possible the usual terminology:

Ranking indicators (RI), or numeral modifiers: *at least*, *at most*, *exactly*

Numerical comparatives (NC): *more than, less than, between ... and ....*

So the labels "numeral modifier" or RI cover, up to now, *at least*, *at most, exactly*, but not *more than*.

Although differing from Landman (2000), the present study has many important features and objectives in common, especially concerning the derivation of existence and maximality claims involved, respectively, by indefinites (or bare numerals) and modified numerals. Our main goal is to propose a model for the existence claim and the maximality claim associated with numeral modifiers. In this respect, apposition offers very interesting data, illustrated by (6) and (7):

(6) She invited at least two persons, Pierre and Jean.

(7) He invited at most two persons, his father and his mother.

Most views of this kind of apposition hold that it needs a previously introduced set, the members of which are (exhaustively) enumerated by the list of appended expressions. Apposition can thus be taken as an argument showing that, in (6), "at least two persons" introduces a set of exactly two persons (the existence of which is thus asserted by the sentence), even though the sentence does not imply that the maximal number of persons she invited is two. But in (7) it just might be the case that he invited nobody. So what is the previously established set which licenses the interpretation of an apposed list involving exactly two persons? We will try to show that these data, as surprising as they may appear at first glance, are straightforwardly predicted by our proposal.

The paper is organized as follows. In section 2, arguments are given for sustaining a "two set" analysis of NPs modified by a numeral modifier. This analysis holds that an NP like "at least *n Ns*" introduces two sets in the discourse representation : a set of cardinality *n* (exactly), and the maximal set of *Ns* satisfying the predicate. These arguments are based on Kadmon's 1987 observation on anaphora and on new data involving apposition.

In section 3, I discuss Kadmon's proposal for deriving the observed effects, and I argue that her analysis based on an underlying syntactic ambiguity and on a specific analysis of plural anaphora, is not without problems and lacks independent support. In section 4, a new analysis is introduced, which derives the "two set" analysis as a direct consequence of the semantics of numeral modifiers themselves.

This proposal takes numeral modifiers as introducing a ranking between the set denoted by the modified NP and the maximal set. DRT representation for the postulated semantics of some examples are given, although the paper does not provide a formal algorithm for deriving the DRSs from the syntax. The last part of this section discusses some specific problems of the semantics of *at most*, and in particular the problem of deriving the sets needed by anaphora and apposition for examples like (7). In this discussion I pay special attention to existential sentences, which, as might be expected, raise special difficulties regarding the existence and maximality claims associated with modified numerals.

2. EXISTENTIAL INTERPRETATION, AND MAXIMALITY

2.1. Numerals, Existence, and Maximality

The analysis of indefinites (*a*) and numerals (*one*, *two*,...) put forward in dynamic frameworks like File-change Semantics and DRT amounts to the following features when applied to a sentence like (8):

(8) I read two novels by Gracq during the holidays.

A) Truth conditions: the intersection set[1] satisfying the noun phrase descriptive content properties and the verb phrase properties, contains at least two members.

B) Dynamics: (exactly) one such set of (exactly) two members, is introduced into the discourse and available for anaphoric links.[2]

The "at least" mention in A is a consequence of the existential interpretation of discourse referents: in DRT, for instance, the corresponding representation has a truthful embedding in a Model each time a set of two Gracq novels read by me is found.

The "exactly" mention in B is strongly supported, for instance, by the fact that successions like (9) are odd if *n* is different from *two.*

(9) I read two novels by Gracq during the holidays. These *n* books were wonderful.

(10) I read two novels by Gracq during the holidays. These *three books were wonderful.

Although this "exactly *n*" interpretation of a pronoun anaphoric to a noun phrase of the form *n N* has been claimed to have exceptions (see Sells 1985), I follow

[1] In this formulation, and in all this paper, I leave aside the relevant distinctions between sets and plural individuals.

[2] An anonymous reviewer suggests that this kind of difference for "*nNs*" meaning between an "exactly *n*" reading, relevant for anaphora, and an "at least *n*" reading, relevant for truth conditions, might support a distinction between the *representation* of a term, and the *interpretation* of a term. I think this is a very fruitful way of interpreting the static/dynamic distinction represented by A/B.

Kadmon (1987) who takes it as the rule for such successions. In this context, the notion of "maximality", or "exhaustivity", will come into play in the following way: it is often understood from (8) that "two novels in all" were read, and hence, that the introduced set is the maximal set in the Model satisfying the conditions considered. But this cannot be a part of the *meaning* of the numeral, because in some contexts, the interpretation of *n Ns* is compatible with the existence of *m Ns* sets, with $m > n$. The classical view is that it is a "no more" implicature that is responsible for the default strengthening of *n* to "*n in all*" (Kadmon 1987). Krifka (1992, 1999) provides an approach in which the content of this implicature is derived in a framework making use of Rooth (1985) notion of alternative. See also Landman (2000).

2.2. Modified Numerals: A Preliminary Typology

One can distinguish two kinds of modifiers in combination with a numeral:

A – Ranking indicators: *at least*, *at most*, *exactly*.

B – Numerical comparatives: *more than, less than, between … and …*.

I will focus on RIs, called here for convenience "numeral modifiers", and point out the features which distinguish them from numerical comparatives.

1) Numeral modifiers are floating expressions. I exemplify this with French:

(11) Au moins deux personnes sont venues. Deux personnes au moins sont venues. Deux personnes sont venues au moins.

At least two persons came. Two persons at least came. Two persons came at least.

2) They can be used is isolation, as exemplified by the following dialogue:

(12) A – David Lewis wrote five books.
B – At least (at most, exactly…).

Numerical comparatives do not float, although they can be used in isolation. In such absolute uses, numeral modifiers are typically preceded by "oui" (yes), while numerical comparatives can only be preceded by "non" (no).

(13) A – David Lewis a écrit cinq livres.
B – Non, (*oui) plus/moins.

(14) A – David Lewis a écrit cinq livres.
B – Oui, (*non) au moins/exactement/au plus.

Although I am convinced that this very strong contrast is a key data for understanding the semantic opposition between RIs and NCs, I will not try to provide a detailed derivation of it in this paper, partly because the focus here is on RIs, not on the contrast RI/NC. I hope nevertheless, that the analysis I give for the semantics of RIs will, at least, help to find less surprising the use of a positive answer in (14).

3) Combinatorial latitude of numeral modifiers.

As noticed by Krifka (1999), numeral modifiers can modify determiners like *some* (see (5) above). In this restricted context, it is hard to call them, strictly speaking, *numeral modifiers*. A closer look reveals that they cannot combine with all determiners.

(15) I have read at most *many books.

(16) I have read at least *no book.

(17) I have eaten at least *nothing.

Numerical comparatives are also ruled out with *many*, but they can combine with negative quantifiers (*less than nothing*).

In principle, it is even possible for RIs to combine with NCs, which is most often taken as a clue that two items do not belong to the same syntactic category,[3] as illustrated by (18) and (19).

(18) He makes at least more than 10,000€.

(19) He makes at least between 10,000 and 15,000€.

RIs can combine with proper names and definite NPs as shown by (20).

(20) I will invite at least John and Mary.

RIs can be used with nominal predicates as in (21).

(21) Mary is at least an ASSOCIATE professor. (Krifka 1999)

All these properties show that the analysis of RIs as functors giving complex determiners when applied to (numeral) determiners is problematic (for similar arguments see Krifka 1999). Such a "complex determiner" analysis might work for numerical comparatives, but we have shown that comparatives and RIs have different properties. The syntactic distribution of RIs indicates that, at least in many occurrences, they take scope over a whole noun phrase, not over a determiner.

[3] The combinability of *x* and *y* is not a *proof* that they do not belong to the same syntactic category, as Ora Matushansky pointed out to me (p.c.), but it is most often taken as an invitation to conjecture that they do not.

Moreover, the semantic analysis of a numeral modifier as forming a complex determiner with a numeral, even in some occurrences, raises many problems listed in Krifka (1999): in this approach, adopted in most classical texts on generalized quantifiers since Barwise & Cooper (1981), the difference between the semantics of *n* and *at least n* is difficult to explain, and it is moreover difficult to explain why *n* generates scalar implicatures, whereas *at least n* does not.

2.3. The Maximalization Effect of RIs

Kadmon (1987) notes the following contrast:

(22) Ten kids walked into the room. They were making an awful lot of noise.

(23) At least ten kids walked into the room. They were making an awful lot of noise.

She observes that:

A. in (22), *they* must refer to a set of ten kids (exactly-FC);

B. in (23) *they* can refer to the set of all the kids who walked into the room even if more than ten did (p. 85). The most prominent, if not the only, reading of (22) is that the set of ten is the set of all kids.

I shall take this duality of readings (exactly n / the maximal set of Ns) for a pronominal anaphora to an RI, to be a direct consequence of the semantics of RIs that I call the "maximalization effect" of RIs.

A simple presentation of the maximalization effect, strongly inspired by Kadmon herself, is roughly as follows.

If one takes anaphoric pronouns as picking up previously introduced sets, a noun phrase like *at least n N*:

1) introduces the maximal set of individuals satisfying the conditions of the sentence;

2) introduces a set of exactly *n* elements;

3) cannot introduce any set of intermediate cardinality, whatever one can imagine about the speaker's mind.

I think that Kadmon is perfectly right about the data. I have just a small divergence with her, although the point is not discussed for itself in her dissertation. She says that what holds for *at least* could be generalized to: *about n CN*, *no more than n CN*, *between n and n CN*, etc., and especially *at most n CN* (p. 91), and she adds *more than n CN* (p. 101). I will try to establish, in contrast, that the maximalization effect is restricted to RIs, and does not hold for numerical comparatives like *more than two* and *between n and n CN.*

2.4. The Maximalization Effect as an Attribute of RIs

The maximalization effect arises precisely when both a set of *n* elements (exactly) and the maximal set are parts of the picture, and no set of intermediate cardinality is. It seems that this is a property of modified numerals, not of numerical comparatives. Compare (24) and (25):

(24) She published at least three papers in *Language.*

(25) She published more than two papers in *Language.*

These sentences have similar truth conditions, but there is an important difference if one looks at the dynamics of the sentences, i.e. their capacity to license anaphoric references in the discourse which follows. Compare (26) and (27):

(26) X published at least three papers in *Language.* They are all in my bibliography.

(27) X published more than two papers in *Language.* They are all in my bibliography.

In (26) we expect either three references, or more. If we have more than three, we infer that the list given is, for the speaker, the exhaustive list of X's papers in *Language*. If the list contains three items, no such inference is warranted. In (27), any list will license the inference that, for the speaker, this list is the exhaustive list of X's papers in *Language* that she is aware of.

The following contrast can be used as a confirmation:

(28) X a publié au moins trois articles dans *Language*. Ils sont tous les trois dans ma bibliographie.
X published at least three papers in *Language*. They are all-def-three in my bibliography.

(29) X a publié plus de deux articles dans *Language*. Ils sont tous les *deux (?trois) dans ma bibliographie.
X published more than two papers in *Language.* They are all-def-*two (?three) in my bibliography.

The relevant fact is that no number will produce a natural succession for (29). This can be taken as evidence that no set of definite cardinality is introduced by numerical comparatives.

One could think that the difference is due to the difference between the "$>$" semantics of *more* (as opposed to the "$\geq$" semantics of *at least*), but this is not the case. Consider for instance the complex *three papers or more*: it looks compositionally like a numerical comparative (at least it contains one), it contains a numeral n, and has a '$\geq n$' semantics.

(30) X published three papers or more in *Language*. They are all in my bibliography.

For many speakers, the pronoun in (30) must refer to the maximal collection of the papers, not to a set of three papers and the test already used in (29) gives the expected result:

(31) X a publié trois articles ou plus dans *Language*. Ils sont tous les
X published three papers or more in *Language.* They are all-def-

*quatre (?trois) dans ma bibliographie.
plur *four (?three) in my bibliography.

2.5. *The Maximalization Effect and Apposition*

Apposition data strongly confirm that modified numerals introduce a set of exactly n elements and the maximal set in the representation. They are not brought up very often in the literature, it seems to me, probably because the analysis of the construction is far from clear. The relevant data are exemplified by sentences like (32) and (33):

(32) There is a woman each Frenchman admires: Marie Curie.

(33) There were two men standing in front of the picture: Pierre and Jean.

Although I do not want to be committed to a particular analysis of this kind of apposition, some properties of the construction will be used as a test.

Consider only cases were the appended material is a list of proper names, and the anchor is a numeral (modified or not) NP. A plausible view of the construction is as follows:

– the first part of the sentence introduces a set in the Discourse Representation.
– the appended list is an exhaustive enumeration of the elements of this set.

This requirement concerning exhaustivity is exemplified by (34):

(34) I invited three persons: *Pierre and Jean.

(34) is ill-formed and cannot be used, even for saying that Pierre and Jean were among the persons I invited. We can thus conclude that the appended list must be an *exhaustive* enumeration of a set introduced in the first part of the sentence. This rather uncontroversial and theory-independent property of apposition is a more

reliable test even than anaphoric data that a set has actually been introduced in a given sentence.

For most speakers, it seems that there is a clear difference between (35) and (36):

(35) I invited more than two persons: Pierre and Jean.

(36) I invited at least two persons: Pierre and Jean.

(36) is good for all speakers, but (35) is awkward for most.[4] If our analysis of apposition licensing is correct, it shows that modified numerals introduce a set of (*exactly*) *n elements*, while numerical comparatives do not.

Apposition by means of a list of more than *n* elements is licensed in both cases. In both cases, it is understood as an enumeration of the maximal set of elements satisfying the conditions of the previous sentence:

(37) I invited more than two persons: Pierre, Jean, Max and David.

(38) I invited at least two persons: Pierre, Jean, Max and David.

It was found that while some speakers express a preference for (37) over (38), no-one judges either as incorrect.

Apposition thus confirms that modified numerals (*at least n*) introduce two sets in the discourse: a set of *n* elements, and the maximal set, while numerical comparatives (*more than n*) introduce only the maximal set.

The fact that this "two sets" interpretation is a specific property of modified numerals (as opposed to numerical comparatives) might indicate that there are two different strategies for stating the extension of the maximal set. Numerical comparatives might be operators on numbers, whereas modifiers might be operators on sets. This suggests that semantics should provide two different analysis for RIs and NCs. The focus of this paper is on RIs, and we will take no position on the analysis of NCs.

Negation offers also sharp contrasts between the two constructions.[5] For space consideration, I can only mention some examples without discussing the question at length. If they are in the syntactic scope of a negation, RIs can only be interpreted with wide scope and exactly *n* readings:

[4] NB: for many speakers, there is no good solution for apposition in (35) and even something like: 'I invited more than two persons: Max, Albert and André' is not fully natural.

[5] I am grateful to an anonymous reviewer for this remark about negation and for a very interesting example:

(i) I invited more than two persons: Pierre and Jean.

(ii) I did not invite more than two persons: Pierre and Jean.

(i) is odd, as already said, but (ii) is fine; this would deserve an explanation, which I cannot go into here for space consideration. Note, as a possible clue, that in (ii), *Pierre and Jean* is interpreted as the maximal set of persons I invited.

(39) I did not invite at least two persons.

This is why the sentence is odd if the kind of objects considered makes a specific interpretation unavailable:

(40) I did not eat at least two cookies this morning.

In contrast, in the same context, NCs can easily be interpreted in the scope of the negation:

(41) I did not eat more than two cookies this morning.

3. DERIVING THE MAXIMALITY EFFECT: KADMON'S PROPOSAL AND ITS PROBLEMS

The main problem Kadmon (1987) tries to solve is the following: suppose *at least n Ns* means "*m Ns*" with $m \geq n$; then any set of *m* members (that the speaker might have in mind) should do for an anaphoric reference; but this is not what happens. What we get as a reference for an anaphoric pronoun, is either a set of *n* elements, or the maximal set.

3.1. Why is the Maximal Set Introduced by Modified Numerals?

- because of the modifier itself?

Kadmon says that any derivation from modifiers themselves would be *ad hoc* and does not correspond to any intuition about the semantics of items like *at least*, or *at most*, etc.

- because of the vagueness induced by the modification of a number?

Kadmon insists that vague indefinites like *some* do not behave this way:

(42) Some friends of mine live in Massachusetts. They play music all night (from Sells 1985).

Does not imply that *all* my friends living in Massachusetts play music all night.

Kadmon concludes roughly as follows: maximality (i.e. the accessibility of the maximal set) is a matter of semantics, not a strict matter of pragmatics; if it were a matter of pragmatics, it would be defeasible in favor of a smaller set, which is not the case. But maximality can be motivated pragmatically: it is in order to satisfy the unicity requirement of definite NPs that we select the only unique collection (i.e. the maximal collection).

Her dilemma is that either one defers the selection of the maximal set to a later mechanism like a uniqueness requirement of definite NPs, or one needs a semantic selection of the maximal set against other correct alternatives, which seems desperately *ad hoc*.

There is, I believe, a way out and will myself conclude with a proposal for generating the maximal set *in situ*, rather than by a later mechanism.

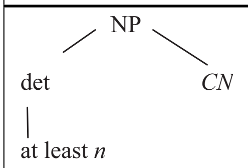
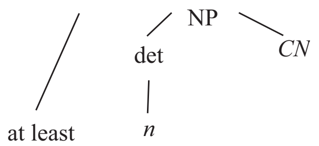
3.2. Why is a Set of (Exactly) n Elements Introduced by Modified Numerals?

Up to this point, Kadmon gives a way of explaining why the maximal set is selected by an anaphora to *at least n CN.* But she has also to explain why a set of exactly *n* members is also made available for an anaphoric pronoun.

Kadmon argues that this is the case because*at least* is syntactically (and hence semantically) ambiguous (p. 102):

- *at least* can be a part of a complex determiner: it only introduces then the maximal set;
- *at least* can have scope over the NP as a whole: the NP then provides a set of *n* elements (exactly).

The following table is a schematic view of Kadmon's solution:

A analysis	*B analysis*
Modifier as building a complex Det	Modifier taking scope over the NP
	
The NP always introduces any set such that $\|X\| \geq n$, including the maximal set.	The NP introduces a set of (exactly) *n* elements as a numeral would do.
Accessible for anaphora: only the maximal set Why? Because of the unicity constraint on definites.	Accessible for anaphora: only a set of *n* elements Why? Only a variable $\|X\| = n$ is introduced
at least n is given a syntax/semantics very close to comparatives (FC).	The semantics of the whole is inherited from an inside indefinite (bare numeral) (FC).

For Kadmon, there is a one to one projection from the different syntactic structures to the different potential antecedents: "The anaphora to a set of exactly *n* members with *at least CN* is allowed iff the structure is [B]" (p. 103). She argues that there are independent arguments in favor of the alleged syntactic ambiguity:

1) *at least* can modify NPs. See (4) and (5).
   For Kadmon, this indicates that the B analysis may be necessary. Note however that this argument does not necessarily support the idea of ambiguity but could

rather show that *at least* can never be analyzed as a part of a complex determiner, or in other words, that the B analysis is sufficient for *at least.*

2) Only items which can modify NPs (i.e. *at least*, *at most*) give rise to the introduction of a set of exactly *n* members.

In Kadmon's text, *at least*, *at most* contrast in this respect with *more than two*, *about three*, *not less than three*, *less than four*, *no more than three*. All the expressions put in contrast to *at least/at most* are what we call here numerical comparatives. We fully agree that NCs do not introduce a set of exactly *n members* (see section 2). Kadmon's observation is thus very close to: "only RIs, and not NCs, can combine with an NP, and can introduce a set of exactly *n*members". This is not an argument in favor of a double analysis of *at least*, but in my view, at first glance, an argument in favor of a different analysis for RIs and NCs. Once a double analysis for RIs is assumed, as in Kadmon's view, the fact that RIs license the *exactly n* set and can combine with NPs does not prove that they license the *exactly n* set iff they combine with an NP.

In other words, Kadmon might be right is assuming that RIs can take different syntactic scope (determiner, NP), but she has no knock-down argument for assuming a one to one correspondence between the different postulated syntactic representations and the contrast maximal set/exactly *n* set.

There are also intrinsic problems with Kadmon's generation of the *exactly n* set.

First of all, a theory postulating a double syntactic analysis for a given lexical item and assuming a correlation between the syntactic structures and the semantic interpretations should be supported by independent syntactic correlates. For instance, it should be plausible to assume that *at least* can only have the A analysis when it immediately precedes the numeral, and cannot when it is in another position. This would lead us to expect that when *at least* is in floated positions, the maximal set cannot be referred to by a plural pronoun. This however is not supported by the facts. Consider (43):

(43) Deux personnes au moins m'ont écrit
'two persons at least me wrote'

It seems that both the *n exactly* set and the maximal set are made accessible by (42) which is not predicted if the floated position is associated with the B analysis. A compelling argument in favor of the one to one correspondence would be a case were the syntactic analysis is, with no doubt, A (or B) and for which only the expected kind of anaphora is licensed. All I can say is that, to my knowledge, no such case has been provided.[6]

[6] An anonymous reviewer brings to attention cases like:
1) At least, I will invite two persons.
2) I will invite two persons, at least.
It seems to me that they license only the *exactly n* interpretation. They would be an argument in favor of Kadmon's view if they are a realization of the B structure. But I am not sure this is so. In this kind of structure, *at least* has scope over the whole sentence, and has no special connection to the interpretation

It can also be observed that this analysis gives more than is needed for generating the interpretations. Consider the B analysis. It seems that its full specification could make accessible all that is needed, and would make the A analysis unnecessary. The B analysis inherits the introduction of a set of exactly *n members* from an "inside" NP of the form *n Ns*, which is what such an NP would do in any case.

At least is then considered as an operator having scope over the NP. It is difficult to state its contribution to the interpretation without stating something like: *at least* asserts that the maximal set satisfying the conditions of the sentence is equal to or greater than n. What I mean here is that once the need for making a set of exactly *n members* a part of the representation is recognized, the consequence that the maximal set is another ingredient follows almost necessarily. If this is true it means that both sets are made accessible, providing all we need. Once the two sets are made accessible, the interpretation of a plural pronoun will be just a matter of choice between two candidates when more than one antecedent is accessible. The need for the A analysis then becomes questionable.

This is precisely the track followed in the rest of this paper. I reject the double analysis of modified numbers and adopt a variant of the B analysis for them.

In this new proposal for modified numerals, both the maximal set and a set of exactly *n members* are made part of the representation and are consequently accessible for anaphora and apposition. No other set is made accessible, which derives straightforwardly Kadmon's observation on the absence of intermediate cardinality sets.

The fact that the maximal set comes into play will be seen as a common semantic property of RIs (*at least*, etc.) and NCs (*more than*, etc.). It is my version of the notion of "maximalization trigger" introduced by Landman (2000).

Although I shall not go into detail here for NCs, I will take them to be operators taking two arguments, the cardinal of the maximal set, and n, and expressing a relation between these two numbers. The only introduced set is the maximal set, whence the observed properties for anaphora and apposition.

In contrast, I will analyze RIs as expressing a ranking relation between two sets: the maximal set and a set of exactly n elements.

4. A PROPOSAL FOR THE SEMANTICS OF "NUMERAL MODIFIERS"

4.1. Some Desiderata for a Proposal

A – Against a syntactic ambiguity as a basis for the two sets analysis.

Contrary to Kadmon's proposal, it would be nice to assume a single syntactic analysis for RIs, and to provide an analysis where the duality *maximal set/exactly n members set* is not a correlate of a syntactic ambiguity. This duality should be

of a given NP. A confirmation of this comes from the fact that this initial/final *at least* can be used with clauses deprived of any argument:
(iii) At least, it rains.

seen as a reflection of the existence of two components in the representation among which anaphora and apposition can choose an antecedent.

B – Numeral modifiers are maximalization triggers.

If the maximal set is not a (semantic) part of the representation of indefinites and numerals (see section 1), the presence of the maximal set with RIs is part of their interpretation. Although some authors like Kadmon find such an approach *ad hoc*, it seems that this conclusion is the only one available, unless the explanation of the anaphoric properties is deferred to a property of definite NPs themselves (as in Kadmon's approach of definite anaphora). Note that Kadmon indeed generates any set of cardinality greater than n, and gives the responsibility of picking up only the maximal one to a property of definite anaphora. It is surely worthwhile trying to avoid an explanation based on another part of the linguistic system. Note that, in addition, this assumption about unicity would have to be extended to apposition, yet another linguistic category.

It will be assumed, then, that the introduction of the maximal set in the representation, and in the semantic calculus of truth conditions, is associated with the semantics of RIs.

C – Numeral modifiers are not determiners (nor determiner modifiers).

The arguments in Krifka (1999) based on the floating nature of these expressions, together with their ability to combine with different syntactic categories, make it plausible to exclude them, like *only*, from the class of determiners or determiner modifiers.

A deeper comparison reveals that RIs have many features in common with *only*, not just the fact that they are not determiners. Like *only*, numeral modifiers need two arguments: a noun phrase, and the whole predication in which the noun phrase is inserted. Consider a sentence and an NP in a sentence corresponding to a set in the semantic representation; a RI having these two elements as arguments states that the set corresponding to the argument is in a certain ranking relation to the maximal set of individuals which satisfies the conditions expressed in the sentence.

D – The same analysis should work for the modification of definite NPs and indefinite ones.

The previous desiderata will help to get a uniform analysis. In the case of a definite NP, the set introduced in the representation by the NP is the set it refers to. The maximal set relevant for the interpretation is the set of all individuals satisfying the predicate.

4.2. The Modification of Definite NPs

RIs can take scope over a definite NP as in (44)

(44) I will invite at least Pierre, Jean and Marie.

The interpretation of (44) is: the whole set of people I will invite includes the set {*Pierre*, *Jean*, *Marie*} The underlying semantics is thus based on the set-theoretic relation "$\supseteq$". But it would be more general to state that *at least* expresses a ranking relation between two sets: the set of all x's such that I will invite x, and the set provided by the NP, i.e. {*Pierre*, *Jean*, *Marie*}. Set inclusion, then, would be a particular case of a more general meaning. Krifka (1999) shows convincingly that a hypothesis of this kind is needed if one wants to take cases like (4) as exemplifying the same lexical item as cases where the modified NP includes a numeral.

In the following DRT representation, I will label the maximal set Σx because it is close to the abstraction operator of Kamp and Reyle (1993). This set is defined as follows: if the scope of *at least* is the definite NP argument A in a sentence $P(A)$, Σx is the set of all x such that $P(x)$ is true. I will assume that the conjunction *Pierre, Jean and Marie* introduces an entity of the same type represented by a capital X. The symbol "$\supseteq$" is to be interpreted as usual. (44) will thus have the representation (45):

(45)
$$\boxed{\begin{array}{l} \text{X, } \Sigma\text{x} \\ \text{X = \{Pierre, Jean and Marie\}} \\ \boxed{\Sigma\text{x : x : I will invite x}} \\ \Sigma\text{x} \supseteq \text{X} \end{array}}$$

This representation gives the correct truth value of the sentence. It is worth comparing the representation (45) with the classical representation of *I will invite Pierre Jean and Marie.* The careful reader will have noticed that the representation of this sentence is not a proper part of (45). The relevant difference is that the condition (*I will invite X*) is not present in (45), as it would be in the representation of *I will invite Pierre*, *Jean and Marie*. Although this paper is mainly an exploration about the correct representations, and does not intend to give a detailed algorithm for deriving these representations, a few words are in order about the underlying analysis of RIs. Basically, RIs will be conceived as functions with two arguments: the NP of the sentence and its VP. In this approach, *at least* is not a function applying to the NP–VP combination, and it is not surprising, then, that the condition *I will invite X*, does not appear as a component of (45). Note that in the present case, adding this condition would just produce a redundant DRS, but it will become clear very soon that doing so would make impossible to provide a general analysis for *at least* and *at most.*

In fact, (45) gives more than what is needed for the anaphoric potentialities of the sentence. As it is, it predicts that both discourse referents can be referred to by a plural pronoun, which is not true, at least for this sentence. All speakers agree that if *they* occurs in the following sentence, it can only pick up the set X.

A possible explanation would be to assume that Σx is very weak in referential force, as compared to a definite NP like *Pierre, Jean and Mary*, and that, as a consequence of this referential inequality, it cannot be picked up by a plural pronoun.

An anonymous reviewer pointed to me that although (45) has correct truth conditions, it looks more close to the quasi-equivalent sentence: "Pierre, Jean and Marie are among the x I will invite". For this reviewer, (45) cannot be taken as the correct representation, one reason being that the Σx set has no syntactic counterpart in the sentence, and possibly because that set is constructed by the interpretation of the pronoun, not by the interpretation of the *at least* sentence. This view expresses very clearly an alternative to the one I follow here: this alternative view considers the Σx set as being: (i) not a part of the meaning of *at least*; (ii) derived by synthesis by the interpretation of the pronoun. I already gave some arguments for not choosing this alternative defended by N. Kadmon. One argument is that in general we need the Σx set not only for interpreting pronouns but also for interpreting appositions. But the main argument is that we need this set for stating what is the semantics of *at least* itself. It is not clear what semantics the alternative view would give to *at least* without making use of the maximal set somewhere. It is true that this Σx set has no syntactic counterpart, but there are other parts of natural language description in which one must assume semantic constituents which are synthesized on the basis of explicit syntactic information.[7] The underlying intuition guiding the present approach is that RIs are some sort of "comparative" operators: the first term of the comparison is a set explicitly introduced in the sentence by an NP, and the second one, the maximal set, is a set synthesized by abstraction over the conditions expressed in the sentence.[8]

An other reviewer of the paper raises an important related question about the two sets analysis. The point is that although we make the assumption that two sets are introduced by the *at least* NP, there is no way to refer back to *both* sets in the next sentence. What the data show is that either the exactly n or the maximal set can be picked up by a pronoun, but not both in the same sentence. In other words, if a pronoun finds one of the sets accessible, another pronoun of the same sentence cannot take the other set as its source. And the same is true if one of these sets is selected at first by an apposition. As suggested by this anonymous reviewer, it might be the case that once one of these sets has been made salient by an apposition or a pronominal anaphora, the other one is no longer salient enough for remaining accessible.[9]

[7] The various operations postulated in order to explain how plural pronouns can find their antecedent in the previous context is a good example of such a case. See for instance the discussion of *abstraction* and *summation* in Kamp and Reyle (1993: 344).

[8] Note that the expressions *at least* and *at most* are built on *superlative* expressions in English, and that their French counterpart is built on lexical items used in comparatives and superlatives. As it is well known, it is difficult to make the semantics of superlative without ressorting to some maximal set.

[9] Note that our analysis creates a special case: if we are right, *one* and only one NP triggers the introduction of *two* discourse referents. A natural hypothesis would be that once selected by a pronoun as a reference to one of these sets, no other anaphoric link can come back to the very same NP for picking up another set. What we have in mind is that anaphora is a relation to a discourse referent introduced by an NP, and that a single NP cannot be the source of two anaphoric chains involving more than one discourse referent.

4.3. The Modification of Numeral NPs

We want to maintain that the semantics is the same set relation between a set of type Σx and a set provided by the modified NP argument. Let us try to keep as close as possible to what is needed for definite arguments. A sentence like (46) would thus have the representation (47):

(46) I will invite at least one person (: Pierre)

(47)
$$\boxed{\begin{array}{l} X,\ \Sigma x \\ \text{person } X \\ \Sigma x\text{:}\ \boxed{\begin{array}{l} x : \text{I will invite } x \\ \qquad \text{person } x \end{array}} \\ |X| = 1 \\ \Sigma x \supseteq X \end{array}}$$

This representation says that the maximal set of persons I will invite includes a set of persons X containing one person.

The way the relevant sets X and Σx are constructed and the relation which is stated between them, makes the representation (47) redundant: in other words, some shorter DRSs would have the same truth conditions.[10]

For instance, the DRSs (48) and (49) are equivalent to (47):

(48)
$$\boxed{\begin{array}{l} X,\ \Sigma x \\ \Sigma x\text{:}\ \boxed{\begin{array}{l} x : \text{I will invite } x \\ \qquad \text{person } x \end{array}} \\ |X| = 1 \\ \Sigma x \supseteq X \end{array}}$$

(49)
$$\boxed{\begin{array}{l} X,\ \Sigma x \\ \text{person } X \\ \Sigma x\text{:}\ \boxed{x : \text{I will invite } x} \\ |X| = 1 \\ \Sigma x \supseteq X \end{array}}$$

But if one wants to preserve the dynamic properties of the representations, it can be shown that some reductions should be avoided.

Consider for instance the kind of simplification illustrated by the DRS (51) for the sentence (50):

(50) I will buy at least two apples.

(51)
$$\boxed{\begin{array}{l} X,\ \Sigma x \\ \text{apple } X \\ \Sigma x\text{:}\ \boxed{\begin{array}{l} x : \text{I will buy } x \\ \textit{x : apple x} \end{array}} \\ |X| = 2 \\ \Sigma x \supseteq X \end{array}}$$

[10] I am grateful to Bart Geurts (p.c.) for his comments on this.

Intuitively one might think that the presence of the condition in italics makes a difference. If present, the sentence would mean: the set of apples I will buy will contain at least two apples. If absent, the sentence would mean: the set of things I will buy contains at least two apples. A closer look reveals that the two versions are strictly equivalent, at least if one only considers the truth conditions of the sentence. The maximal set of things I will buy contains two apples iff the maximal set of apples I will buy contains two apples.

But if one considers the potential for anaphoric reference created by the sentence, things look different. For most speakers, it seems that if they can interpret a plural pronoun as a reference to the maximal set in (50), they can only interpret this set as a set of apples. This is an indication that the representation (49) is not the kind of representation we need for capturing both the truth conditions and the dynamic properties of the expression.

But these data based on anaphora do not help for choosing between (48) and (47) since although the condition *person (x)* is not present in (48) the relation of inclusion implies that X is a set of persons, and that X is a set of entities that I have invited.

Nevertheless, a case like (49) shows that it can be necessary to consider a redundant DRS as a representation which is needed for dynamic reasons. In other words, this provides an argument that redundancy is not by itself an argument that a semantic representation is inadequate.

For reasons that will become clear soon, when discussing *at most,* I suggest that the correct representation for (47) is the redundant representation (52):

(52)

$$\begin{array}{|l|}\hline X,\ \Sigma x \\ \text{person } X \\ \Sigma x\text{:}\ \boxed{\begin{array}{l} x : \text{I will invite } x \\ \qquad\quad \text{person } x \end{array}} \\ |X| = 1 \\ \Sigma x \supseteq X \\ \hline \end{array}$$

This paper does not provide a formalized derivation of the postulated representation from the syntactic structure of the *at least* sentences; this task must be deferred to further works. Moreover, the semantics of the DRS, especially regarding the discourse referent Σx, is not formalized in this paper. The reader should thus take the provided DRS as an intuitive illustrations in favor of a new analysis. The main focus here is to argue for a plausible strategy and some general principles for a formal derivation of the representation.

The general strategy is that for deriving an *at least* sentence analyzed as (*at least* $(X)_{NP}$VP), one derives first a set X constrained by the NP descriptive content, then a maximal set Σx by abstraction over the conditions expressed by the NP and the VP, and then asserts the relation $\Sigma x \supseteq X$.

For definite NPs, abstraction returns the maximal set of individuals satisfying VP, and for numeral NPs, abstraction returns the maximal set of individuals satisfying VP and the conditions expressed by the NP.

Let us consider the way an apposition like *Pierre*, is interpreted in (46). Apposition is licensed by the presence of a set in the representation of the sentence. I will not discuss in detail the constraints on the form under which this set must be introduced in order for apposition to be licensed. Proper names, or conjunctions of them do not license apposition, while definite NPs having a lexical descriptive content and indefinites do license apposition. Since RIs share with definite and indefinite NPs the property of introducing sets, it is expected that they will license apposition, and they do.

The proposal provides two such sets: X, the standard representation of the NP (I will call it the reference set), and the maximal set, Σx. If apposition must be interpreted as the *exhaustive* enumeration of a previously introduced set, the set of cardinality 1 must have been introduced by (46) for *Pierre* to be interpreted. If one takes seriously the idea that it should be possible to know whether an enumeration is exhaustive or not, it predicts that apposition to the set having a precise cardinality will be strongly preferred, which seems to be the case.

In the particular example (46), where there is a potential contrast between singular (the cardinality of X is 1) and plural (the maximal set can be bigger), we note that apposition to the bigger (hence plural) set is impossible, and anaphora is very odd, as illustrated by (53) and (54):

(53) I will invite at least one person : *your parents.

(54) I invited at least one person. ?They were sitting here.

I think that it is a special case, due to the fact that the reference set and the maximal set are (possibly) of different types (atom/plural individual). What we observe is that in such cases, only the reference set is accessible for apposition and anaphora. Note that this could be taken as an argument for the presence of an *exactly n* set in the representation of *at least* sentences, and for the focalization of this set over the maximal set, even if one has no real explanation for the constraint being so strong that the maximal set is not accessible.

In the general case (both sets being of cardinality greater than one), our prediction is that both antecedents are accessible for apposition, with a clear preference for the set of definite cardinality (i.e. X, the reference-set), especially for apposition appending a list with a final conclusive tone, and no items like "etc.". In those cases, the matching between the cardinalities imposes the interpretation that the reference-set is being enumerated. In contrast, in cases where a list is appended with such explicit markings of non-exhaustiveness, it is the maximal set which is preferred.

(55) I met at least five people: Pierre, Nicolas, etc.

What apposition reveals in cases like (46) above is that it is a case of a specific interpretation of *one N*, the speaker having in mind an individual (identified by apposition), and asserting that the maximal set, whatever it is, will include this individual. What one has in this case, is the composition of an indefinite interpretation (assertion of existence, introduction of an individual satisfying the conditions) and an explicit statement that this individual might not be unique.

But it is not true that the use of (46) need be a specific one. One can use (46) with no individual in mind, just as a way of stating the number of persons one wants to invite. This duality (specific/non-specific) exists also for indefinites and numerals and is not a problem for this particular analysis of modified numerals. One could just say that if the small set is not specific, then the modified numeral will amount to a mere cardinality relation: the sentence just says that the number of individuals satisfying the predicate is greater than or equal to n.

For anaphora potentialities, we predict that in the general case, both the reference set and the maximal set are accessible. The fact that, if the reference set is a specific reference, it takes priority over the maximal set seems rather natural. In a succession like (56) there is a preference for interpreting *they* as referring to a set of two persons, especially if you think that the speaker has two specific persons in mind.

(56) I will invite at least two persons. I will phone them tomorrow.

We do not exclude that the maximal set can be accessible for an anaphoric reference.

Questions following such expressions are ambiguous:

(57) A. I met at least two colleagues in this workshop.
B. Who?

Even if one thinks that an answer must be exhaustive, it is difficult to decide whether B is asking about this set of two persons, or about the set of all colleagues A met. Again, the notion of exhaustivity might lead to preferring an interpretation of the question as being about the reference-set (B would then be able to see that the answer is exhaustive), but the other option is open as well (B might want to get the list of all colleagues A met).

4.4. Monotonicity and Existence Claim

Let us try to apply mechanically what has been done for *at least* to a decreasing operator like *at most*. What we want to preserve is that the sentence expresses a relation between two sets, X and Σx, Σx being derived by abstraction over the conditions of the sentence. To say that *at most* is a decreasing operator means precisely that the maximal set is stated to be included into another set, and possibly null.

When applied to (58) the derivation rules used up to now would product (59):

(58) I will invite at most Pierre, Jean and Marie.

(59)
$$\boxed{\begin{array}{l} X,\ \Sigma x \\ X = \{\text{Pierre, Jean and Marie}\} \\ \Sigma x : \boxed{x : \text{I will invite } x} \\ X \supseteq \Sigma x \end{array}}$$

The only change we have made is a permutation of X and Σx for $\supseteq$.

What the representation (59) says is that the maximal set of x I will invite is a subset of X, X being the set {Pierre, Jean, Marie}.

There are at least one new problem. The sentence (58) is compatible with a no-invitation situation. This could be seen as contradicting the occurrence of a variable for Σx at the top level of the representation, which implies in classical DRT that the set exists. For sets in the scope of a decreasing operator like X $\supseteq$ Σx, any theory will have to assume some sort of conditionalizing ("I introduce Σx, if this set is not empty, which might be the case...") As a mere notation of sets so specified, I will use $*\Sigma x$.[11]

Note that the key-decision is the decision to constrain X without asking that it satisfies the VP conditions (see section 4.2). As long as one considers an increasing operator like *at least*, it may just look as a matter of redundancy. But the possibility that the set of invited x be smaller that X is not compatible with a specification of X as a set of invited persons. If X is an existing set of invited persons, the maximal set of invited persons cannot be smaller than X, it can only be equal or greater. So if one considers a decreasing operator like *at most*, this decision is what makes the representation strictly parallel to the representation of *at least* sentences, and moreover correct for truth conditions and dynamic properties.

When applied to numeral NPs as in (60), this simple algorithm predicts that (60) is correctly represented by a DRS like (61):

(60) I will invite at most one person (: Pierre)

(61)
$$\boxed{\begin{array}{l} X,\ *\Sigma x \\ \text{person } X \\ *\Sigma x\text{:} \boxed{\begin{array}{l} x : \text{I will invite } x \\ \quad x : \text{person } x \end{array}} \\ |X| = 1 \\ X \supseteq *\Sigma x \end{array}}$$

What (60) shows is that although the maximal set might be empty, the existence of a set having the specified cardinality (1) is entailed by the sentence, since apposition

[11] As pointed to me by an anonymous reader, the semantics of the variable Σ, an extension of classical DRT of Kamp and Reyle (1993), would remain to be done explicitly. One possibility would be to consider this variable as covering sets of any cardinality (including the null set) which would dispense of distinguishing a starred version. The specific feature of the starred version is that the speaker is associating properties to a discourse referent in case such discourse referent exists.

is licensed. If apposition works as usual, a set has been introduced, and *Pierre* is the exhaustive enumeration of the elements of this set. Of course, this set is not *the set of persons I will invite*, since the modifier explicitly says that this number might be zero. This set is specified in the representation just as a set of exactly one person.

Note that the set made accessible for apposition *must* be a set satisfying the condition introduced by the numeral NP. The following sentence, for instance, is very odd:

(62) *I will represent at most one person in the picture: a table

This indicates that the computation of the reference-set X for the decreasing operator *at most* preserves, at least in cases we are examining, the nominal content of the NP. The empirical counterparts of this assumption are the constraints on apposition illustrated in (62), and more generally the content of the existence claim since in the theoretical framework we are working with, the set X is assumed to exist in the Model. In the next section, we will discuss some special cases.

In cases like (60) it can be the case that the speaker, when saying that she will invite at most one person, has a specific person in mind, say *Pierre.* Apposition corresponds to this case, and is interpreted as an enumeration of this set.

But it can also be the case that the speaker has no specific referent set in mind. In this case, it is the cardinality of the set X, which is in focus, not the identity of its members. These two cases are illustrated respectively by (63) and (64):

(63) I will invite at most two persons, my parents.

(64) I will eat at most three cookies this morning.

In the case of a non-specific reading of X, highly preferred in (64), the interpretation is very close to the corresponding numerical comparative. For a comparison between numerical comparatives and RIs, see Corblin (2006).

4.5. The Problem of (*Some*?) *Existential Sentences*

Consider sentences like:

(65) There are at most three solutions to this problem.[12]

It is not obvious that the strategy which gives good results for (60) will be enough here. The main problem, of course, is that if we mechanically transpose the analysis, the sentence should claim that there is a set of three solutions, and at the same time, it should make the claim that there is zero, one, two, or three solutions to this problem.

[12] I am grateful to Gennaro Chierchia for having pointed out to me that these sentences deserve special attention.

The main point, it seems, is that (65) does not claim there are three solutions to the problem. But note that one still needs to assume that a set of cardinality 3 is introduced in the sentence to accommodate apposition, as (66) shows:

(66) There are at most three solutions: leaving, asking for an explanation, and fighting.

Any other test previously used in this paper, leads to the conclusion that (66) claims that there is a set of cardinality 3, and introduces this set in the discourse representation.

I begin by indicating what a correct representation might be, and then make suggestions as to why and how this representation can be obtained. At first glance, what a sentence like (65) says and makes available for the following discourse is something like (67):

(67)

$$\boxed{\begin{array}{l} \mathrm{X}, *\Sigma \mathrm{x} \\ |\mathrm{X}| = 3 \\ {}^{*}\Sigma \mathrm{x} : \boxed{\begin{array}{l} \mathrm{x} : \text{is a solution} \\ \quad \text{to this problem x} \end{array}} \\ \mathrm{X} \supseteq *\Sigma \mathrm{x} \\ \\ \end{array}}$$

Roughly speaking, (67) means: there is a set of three entities such that the maximal set of solutions to this problem, if there is a solution, is contained in this set.

This correctly gives the interpretation of apposition in (66): the three actions mentioned are the set of three “things” among which the maximal set of solutions, if there is any solution, is to be found. This might also give the correct solution for anaphora in sentences like (68):

(68) There are at most three solutions to this problem. I will present them successively.

In (68), it seems that there are two interpretations:

A. I will present the three things among which is a solution, if there is any. X is the antecedent of *them*.

B. I will present the maximal set of solutions. ($^{*}\Sigma$x) is the antecedent of *them*.

There is a strong preference for the A interpretation (anaphora to the reference-set) which can be justified by features we have already mentioned: *they*, in principle, requires a plural referent, but $^{*}\Sigma$x is not necessarily a set of cardinality greater than one (and possibly does not exist), while X exists and is plural.

If this representation of (65) is accepted, we should try to explain why existential sentences trigger this special mapping from the syntactic structure to the semantic components of the DRS. But first, we need to state the peculiarities of this mapping more precisely. What obtains is that the content of the NP, which is otherwise a constraint on the existing set X, is not, this set X being left unconstrained except for cardinality.

How can this mapping be correlated to the very notion of existential sentences? I have no detailed and fully motivated answer to this question because the semantics of existential sentences is a notoriously difficult issue. I will only make some suggestions. A simple indefinite sentences like "There are three solutions" is taken to be, in the framework we are working with, the mere existence claim of a set of (at least) three solutions, something like (69):

(69) There are three solutions

X $\|X\| = 3$ solution (X)

Roughly speaking, this is distinguished from non-existential sentences like *a man came in* by the fact that the content of the noun phrase in existential sentences, is not intersected with other properties stemming from the VP. This is a way of saying that "is" transmits no semantic information to the DRS (69).

If one prefers to give a representation to the verb, as one could try to do for a sentence like (70):

(70) Three solutions exist

the representation (71) might be proposed:

(71)

X $\|X\| = 3$ solution (X) exist (X)

But, obviously, this is a very odd DRS, because the last condition, so to speak, "expresses" the existential semantics which defines the interpretation of variables. It might be seen as ill-formed per se, because it would be verified if there is a set X such that X exists. So if one wants it as a condition in order to be compositional, one must provide a special way of interpreting this condition; it will have as a result that this condition has no semantic content, which makes (71) and (69) very close. Let us consider, then, that (69) is the correct representation.

The construction algorithm of the *at least/at most* DRSs sketched up to now works as follows:

1) It builds the reference-set X as the set constrained by the NP content, i.e. a set of cardinality *n* of entities of category *c*, where *c* is the descriptive content of the NP. For (65) this gives a set of 3 solutions.

2) It builds the maximal set Σx of entities satisfying the conjunction of the conditions expressed by the NP and by the VP. For (65) it gives the set of all solutions to this problem.
3) It states that Σx, if it exists, is contained in X.

One must first observe that for some existential sentences, this algorithm provide correct results. Consider for instance (72):

(72) There are two men in the garden.

The standard representation is (73):

(73)

X Men (X) In the garden (X) $\lvert X \rvert = 2$

Consider now (74):

(74) There are at most two men in the garden.

The proposal will derive the following representation:

(75)

<table><tr><td>X, $*\Sigma x$
Men (X)
$\lvert X \rvert = 2$
$*\Sigma x$: <table><tr><td>x : In the garden x
Man x</td></tr></table>
$X \supseteq *\Sigma x$</td></tr></table>

The existence claim is limited to a set of two men, and this representation derives correctly apposition, as well as anaphora data of (76) and (77):

(76) There was at most two men in the garden: her father and his brother.

(77) There was at most two men in the garden. I know them both.

As already said, $*\Sigma x$ is not the best candidate for apposition or anaphora, in our view because of its very nature (possibly non-existent). But apposition in (76), and anaphora in (77) are predicted once a set of two men, possibly not is the garden, is made available, which (75) provides, and the difficulty to have access to $*\Sigma x$ can be explained just as a matter of concurrence between the two sets.

If analyzed this way, existential sentences are not per se an exception to the general principles governing the computation of the reference-set X, but are perfectly in line with the general case.

A consequence of the proposal is that, in the general case, any *at most n N* sentence generates an existence claim for an *at least n N* set.[13] Let us consider examples like

(78) There are only three women in the island. So I will invite at most three women.

This succession is perfectly natural. It would not be the case for any *n* in the second sentence if *n* is greater than three.

If the problem with sentences like (65) is not triggered by existential constructions as such, it can only be triggered by the specific semantic properties of the lexical material involved, namely *solutions to this problem.*

I will briefly consider some lines of investigations.

A solution suggested to me by an anonymous reviewer is to consider that there is no problem at all. The general idea is that the content of the NP is *solution,* and the abstraction operator builds the maximal set of solutions to this problem. It would be perfectly correct then to maintain that the sentence asserts that there are three solutions, because to assert that something is a solution does not imply that it is a solution to this problem. This gives the maximal parallelism between *There are at most three men in the garden* and *There are at most three solutions to this problem.* Both would directly be derived by our proposal.

But I do not feel entirely convinced by this line of explanation, because there is a strong intuition that it is impossible to see something as a solution without saying of which problem it is a solution. In that case, the problem with the general algorithm would be that it would have to split and compute separately two inseparable parts of a constituent. This is one might call the "relativity" problem.

Another relevant property of the semantics of the considered lexical items is that there is a strong implicature that some problems have no solution, which could be called the "existence" problem.

It seems that these two problems can play a role for explaining why lexical items like *solutions* in existential *at most* sentences, are not directly (or not uncontroversially) derived by the regular algorithm introduced in this paper, but I have no detailed explanation to offer for this.

5. CONCLUSIONS

5.1. Summary of the Proposal

RIs (*at least*, *at most*) are seen as operators having scope over NPs. They are analyzed as ways of stating the cardinality of a set Σx by a comparison to a set provided by the NP, the reference-set X. This reference-set X plays the role of a standard

[13] Krifka (1999) says that *at most n N*-VP generates in many cases the *presupposition*: *One N at least*- VP. This is a very different matter that we cannot discuss here for space consideration.

of comparison for evaluating the maximal set Σx by way of set relations. The existence of the reference set is entailed by the sentence.

The maximal set Σx introduced in the representation by RIs is the set of entities satisfying the conjunction of the conditions expressed by the NP and the VP and is derived by abstraction. The reference-set X constrained by the lexical content of the sole NP.

The proposal is intended to deliver both the truth conditions of sentences containing RIs and the data regarding apposition and anaphora to such sentences. If the underlying reference-set X has a specific interpretation, this gives rise to readings in which the set inclusion interpretation is prominent and in which apposition is licensed. In case the underlying reference-set does not receive a specific interpretation, this gives rise to readings which specify only the cardinality of the maximal set. The maximalization effect of RIs, is thus triggered by the category RI, not by the semantics of individual items like *at least*, *at most*, etc. The proposal is not intended to cover numerical comparatives (e.g. *more than n*) and holds that there is a strong contrast between the two categories of forms.

5.2. Links towards Other Approaches

Kadmon (1987) is to my knowledge the only approach to RIs focusing on their potential for anaphoric references. This study takes Kadmon's insight as a starting point and adds new data about apposition which confirm them. I give some arguments in the paper for proposing a different analysis, and I hope that the reader can now make up her mind. The main features of my proposal contrasting with Kadmon's are: two sets are generated by the semantics of RI, and the sets are not correlated with a structural ambiguity. In my proposal, both sets are parts of the representation for any use, while in Kadmon's approach, only one of them can be. It is difficult to find conclusive empirical evidences establishing that the two sets are both available or establishing that they are not. In principle, one of them, say X, should be taken as an antecedent by an apposition while the other one, the maximal set, should be the antecedent of an anaphoric pronoun. But many factors complicate the picture: for instance, once chosen as an antecedent for apposition, a discourse referent is made much more salient, which makes the accessibility of the other one less likely for anaphora.

Krifka (1999) is not concerned with the dynamic properties of RIs, but with a compositional semantics deriving the truth conditions of RIs and NCs. My proposal shares with his the view of RIs as having scope over NPs, not as combining with a numeral to produce a determiner, and the basic intuition that RIs and particles like *only* looks alike on many respects. I think nevertheless that a detailed comparison is very difficult because Krifka introduces in the course of his paper many innovations which makes his theoretical framework much more sophisticated than the classical conception of semantics used in my proposal. Krifka formulates his analysis of RIs in an extended version of Rooth (1985) alternative semantics, which was first applied to the semantics of *only.* It is thus interesting to make some suggestions about a possible extension to *only* of an analysis making use of a notion of maximality related to the one used in this paper. The analysis of *only* is far beyond the scope of this paper, but since we noticed similarities between RIs and *only*, one might think that we should have to give some more substance to this observation. Consider the example:

(79) Only John works

At first glance, a plausible analysis of (79) is something like: the set {*John*} is the maximal set such that *x works.* Leaving all details aside regarding the way the two sets are computed, and considering that the representation is given under this form, it seems that the meaning of *only* in terms of alternatives is easily deduced: if the relevant (possibly contextual) domain of discourse contains, say Mary, the representation of the sentence implies that Mary do not work. Note that if *only* is analyzed as an *identity* between a reference-set and the maximal set two consequences follow: (i) a similarity of interpretation with the RI *exactly n* is predicted, which seems observed, at least in some contexts; (ii) there is no way to distinguish the two sets, for anaphora, for instance. These few remarks are of course only tentative; they are just a way of showing that the analysis we propose for RIs do not prevent to capture the observations sustaining the intuition that RIs and *only* have common properties.

Landman (2000) is another approach mainly concerned with the compositional derivation of RIs and NCs truth conditions and implicatures. An important difference is that Landman takes RIs as combining with the numerals and give a great importance in his proposal to the scalar properties of numerals. But since he derives what he calls the "existence claims" and the "maximality claims" at the event type level, and because he takes the maximality claim to be a property of RIs, there are nevertheless many points of convergence with the present proposal. Given the ambition and the complexity of Landman's work (see especially Landman 2000, Lecture 7) it is very difficult, and possibly hopeless, to make a detailed comparison with the present proposal in a limited space, and I let this task for further works.

A recent work by Geurts and Nouwen (2005) is based on the the assumption that *at least* and *at most* embodies modal operators. A detailed comparison with this very different approach will remain to be done.

REFERENCES

Barwise, J. & R. Cooper (1981) "Generalized quantifiers and natural language", *Linguistics and Philosophy*, 4, pp. 159–219.

Corblin, F. (1997) "Les indéfinis: variables et quantificateurs", *Langue Française*, 116, pp. 8–32.

Corblin, F. (2002) *Représentation du discours et sémantique formelle: Introduction et applications au français*, Collection Linguistique nouvelle, Paris: P.U.F.

Corblin, F. (2006) "The semantics of Paranumerals" dans. In L. Tasmowski & S. Vogeleer (eds.) *Non-definiteness and Plurality*, Benjamins, Linguistics Today Series, pp. 331–353.

Geurts, B. & R. Nouwen (2005) "*At least* et al." Ms. University of Nijmegen.

Heim, I. (1982) *The Semantics of Definite and Indefinite Noun Phrases*, Amherst: University of Massachusetts.

Kadmon, N. (1987) *On Unique and Non-Unique Reference and Asymmetric Quantification*, Ph.D. dissertation, Amherst: University of Massachusetts.

Kamp, H. & U. Reyle (1993) *From Discourse to Logic*, Dordrecht: Kluwer.

Keenan, E. & J. Stavi (1986) "A semantic characterization of natural language determiners", *Linguistics and Philosophy*, 9, pp. 254–326.

Krifka, M. (1992) "A compositional semantics for multiple focus construction". In J. Jacobs (ed.) *Informationstruktur und Grammatik*. 17–53. Opladen: Werdeutscher Verlag.

Krifka, M. (1999) "At least some determiners aren't determiners", In K. Turner (ed.). The Semantics–Pragmatics Interface from Different Points of View, Elsevier, pp. 257–291.

Landman, F. (2000) *Events and Plurality: The Jerusalem Lectures*, Dordrecht: Kluwer.

Rooth, M. (1985) *Association with Focus*, Ph.D. dissertation, Amherst.

Sells, P. (1985) *Restrictive and Non Restrictive Modification*, Report 85-28, CSLI Stanford.